Dada la aplicación lineal f: $M_2(\mathbb{R}) \longrightarrow P_2(\mathbb{R})$ dadas por:

$$f\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = (a_{11} + a_{12}) + 2a_{21}x - 3a_{22}x^2$$

Calcular la matriz asociada a f respecto de las bases

$$B = \left\{\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}\right\} \text{ y } B' = \{1 + x + x^2, -x^2, -x + x^2\}$$

de $M_2(\mathbb{R})$ y $P_2(\mathbb{R})$, respectivamente. Comprobar la relación entre la matriz asociada a f respecto de las bases dadas y la matriz asociada a f respecto de las bases canónicas de los espacios vectoriales dominio y codominio de f.

$$f\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} = \underline{4x} = a(1+x+x^2) + b(-x^2) + c(-x+x^2)$$

$$= \underline{a + (a-c)x + (a-b+c)x^2}$$

$$\left.\begin{array}{r} a = 0 \\ a - c = 4 \\ a - b + c = 0 \end{array}\right\} \begin{array}{l} \longrightarrow a = 0 \\ \longrightarrow c = -4 \\ \longrightarrow b = c = -4 \end{array} \qquad f\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} = (0, -4, -4)_{B'}$$

$$f\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} = -1 = a + (a-c)x + (a-b+c)x^2$$

$$\left.\begin{array}{r} a = -1 \\ a - c = 0 \\ a - b + c = 0 \end{array}\right\} \begin{array}{l} \longrightarrow a = -1 \\ \longrightarrow c = -1 \\ \longrightarrow b = -2 \end{array} \qquad f\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} = (-1, -2, -1)_{B'}$$

$$f\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = 1 - 6x^2$$

$$\left.\begin{array}{r} a = 1 \\ a - c = 0 \\ a - b + c = -6 \end{array}\right\} \begin{array}{l} \longrightarrow a = 1 \\ \longrightarrow c = 1 \\ \longrightarrow b = 8 \end{array} \qquad f\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = (1, 8, 1)_{B'}$$

$$f\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} = 6x^2 = a + (a-c)x + (a-b+c)x^2$$

$$\left.\begin{array}{r} a = 0 \\ a - c = 0 \\ a - b + c = 6 \end{array}\right| \begin{array}{l} \longrightarrow a = 0 \\ \longrightarrow c = a = 0 \\ \longrightarrow b = -6 \end{array} \qquad f\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} = (0, -6, 0)$$

$$C = M_{B,B'}(f) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ -4 & -2 & 8 & -6 \\ -4 & -1 & 1 & 0 \end{pmatrix}$$

$$B_c = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

$$B_c' = \{ 1, x, x^2 \}$$

$$f\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = 1 \equiv (1, 0, 0)_{B_c'}$$

$$f\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = 1 \equiv (1, 0, 0)_{B_c'} \qquad A = M_{B_c B_c'}(f) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$f\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = 2x \equiv (0, 2, 0)_{B_c'}$$

$$f\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = -3x^2 \equiv (0, 0, -3)_{B_c'}$$

A y C son matrices equivalentes, es decir,

$\exists Q, P$ matrices regulares tq $C = Q^{-1}AP$

P cambio de base de B a B_c

Q " " " " B' a B_c'

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix} \qquad Q = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$|Q| = -1$$

$$\mathrm{Adj}(Q) = \left(\alpha_{ij} \right) \qquad \alpha_{ij} = (-1)^{i+j} \, \underline{Q_{ij}}$$

$$\mathrm{Adj}(Q) = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \mathrm{Adj}(Q)^t = \begin{pmatrix} -1 & 0 & 0 \\ -2 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$Q^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$Q^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & -2 & 3 \\ 1 & 1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & 1 & 0 \\ -4 & -2 & 8 & 6 \\ -4 & -1 & 1 & 0 \end{pmatrix}}_{= C}$$